A continuous-time model for scheduling gantry cranes in storage yards

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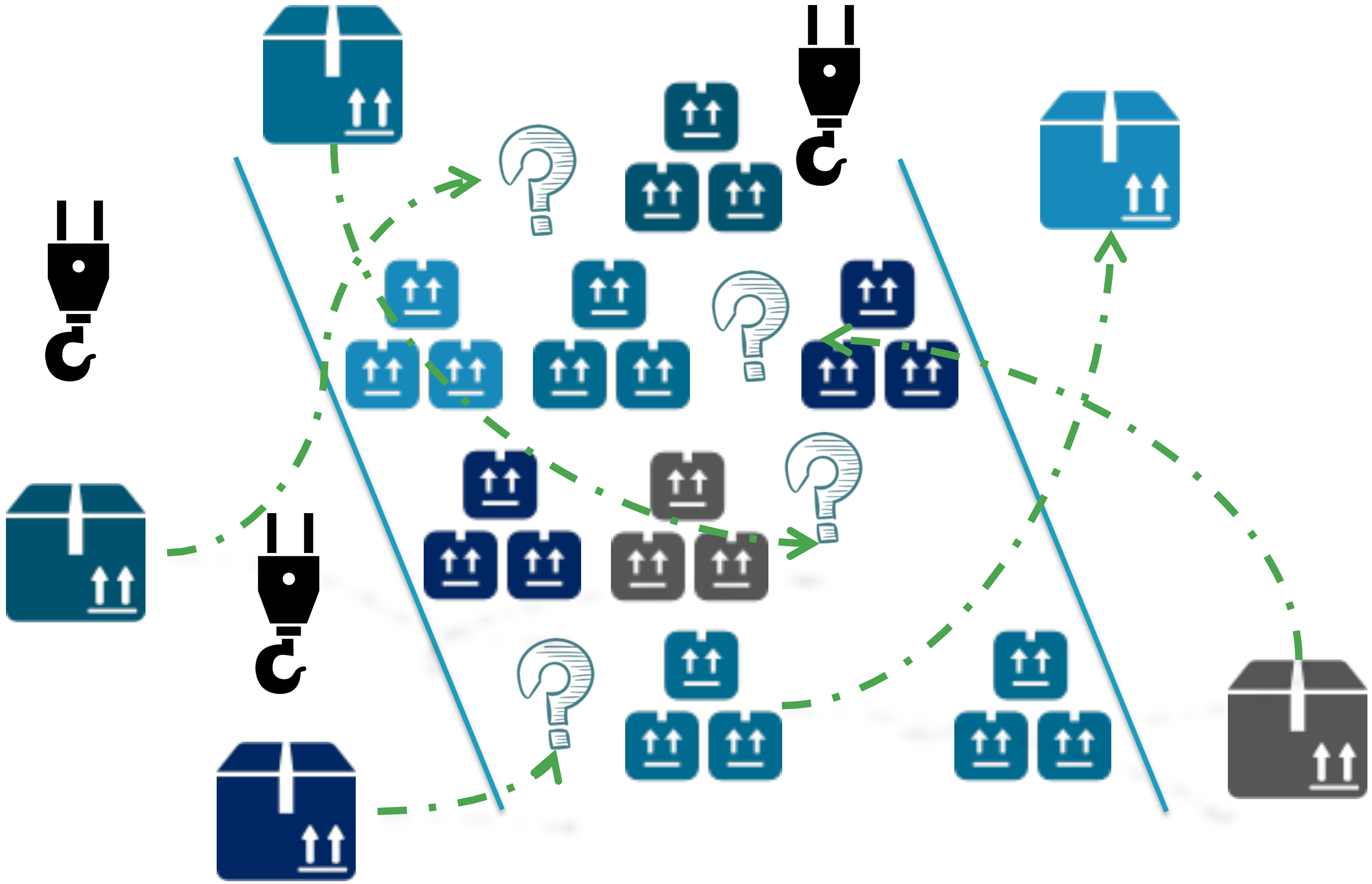
Handling operations in automated warehouses



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Literature overview

- ➢ Automated Storage/Retrieval Systems (AS/RS)
- ➢ Landside crane scheduling in Container Terminals (CT)

System Characteristics

- Crane
  - Number of cranes
  - Movement
  - Crossing is possible?
- Handling
  - Picking / Placing
  - Loads
- Storage area
  - Rack/Yard
  - I/O position
  - Stacking rules

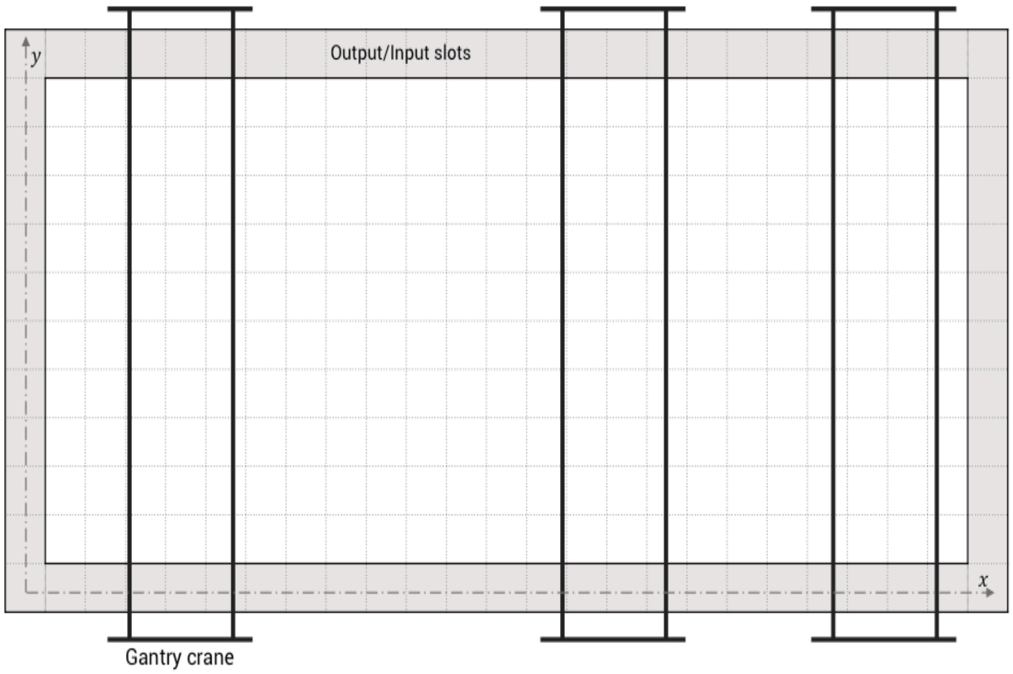
Decision Problems

- Storage assignment
  - Storage strategy
  - Zone sizing
  - Zone positioning
- Sequencing
  - Sequencing method
- Dwell-point of the cranes

System characteristics

Storage yard

- Storage blocks
  - Capacity of one product
  - Stacking level
- Input / Output points
  - Located around the yard



Yard top view

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System characteristics

Rail mounted gantry cranes

- Rail Mounted Gantry Cranes (RMGC) work in parallel
- Dynamic working area per crane
- Cranes cannot pass each other and must respect a safety distance
- One product at a time

System characteristics

Products

Input product

- Origin point
- Destination block
- Operating crane
- Operating order

Output product

- Origin point
- Destination block
- Operating crane
- Operating order
- Precedence movements

Yard product

- Origin point
- Destination block
- Operating crane
- Operating order

Continuous-time model

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Decision variables

$$x_{pm} = \begin{cases} 1 & \text{if product } p \text{ is assigned to block } m \\ 0 & \text{otherwise} \end{cases}$$

$$y_{pc} = \begin{cases} 1 & \text{if product } p \text{ is handled by crane } c \\ 0 & \text{otherwise} \end{cases}$$

$$z_{pq} = \begin{cases} 1 & \text{if product } p \text{ is the neighbour of product } q \\ 0 & \text{otherwise} \end{cases}$$

$$o_{pq} = \begin{cases} 1 & \text{if there is a movement conflict of products } p \text{ and } q \\ 0 & \text{otherwise} \end{cases}$$

$$n_{pq} = \begin{cases} 1 & \text{if } s_q \text{ is greater than end time of movement product } p \\ 0 & \text{otherwise} \end{cases}$$

s_p is the start time of the movement product p

lt_p is the delivery lateness of product p

Handling operations in storage yards

Objective:

Storage cost

- Minimize **storage cost**
  - Assigned block cost
  - Proximity cost

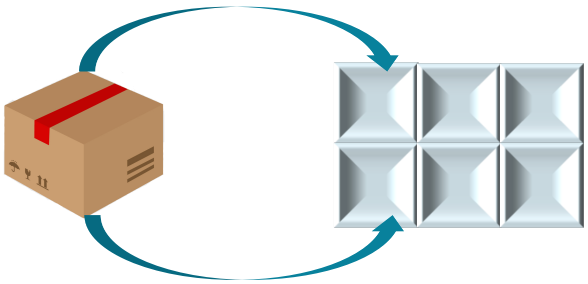
Tardiness

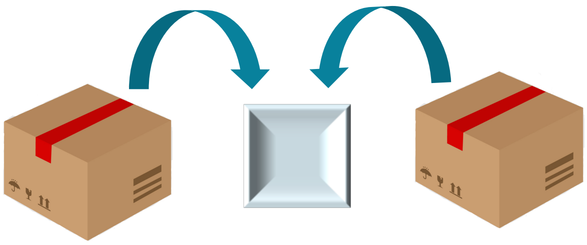
- Minimize **tardiness**

Objective Function

$$Minimize \quad \alpha \sum_{p \in \mathcal{P}_{in}} \sum_{m \in \mathcal{B}} PBC_{pm} x_{pm} \quad + \beta \sum_{p \in \mathcal{P}_i} \sum_{q \in \mathcal{P}} PPC_{pq} z_{pq} \quad + \gamma \sum_{p \in \mathcal{P}_i} lt_p$$

Assignment Constraints

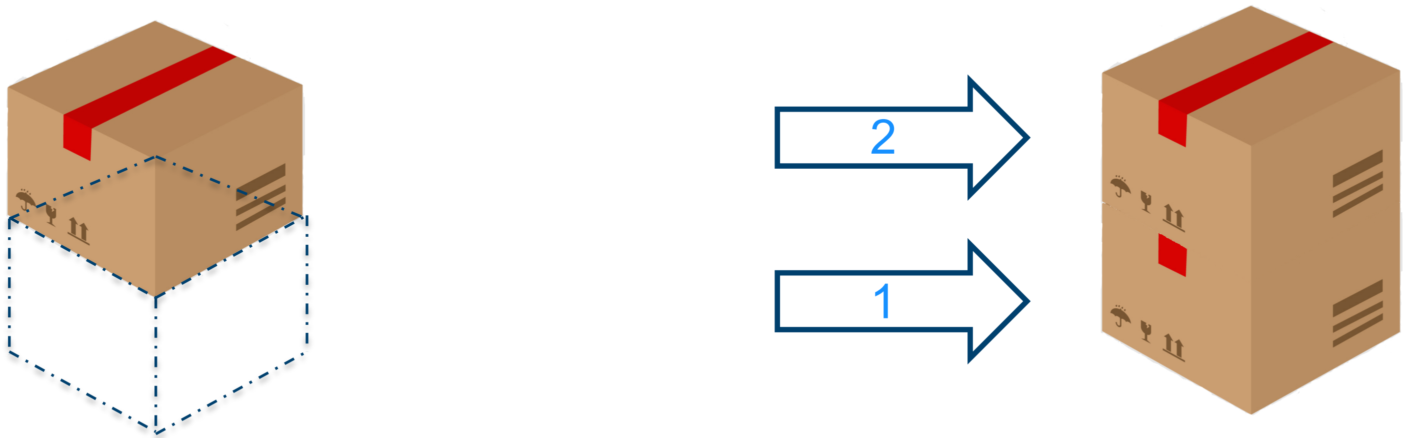




$$\sum_{m \in \mathcal{B}} x_{pm} = 1 \qquad \forall p \in \mathcal{P}_i$$

$$\sum_{p \in \mathcal{P}_i} x_{pm} \leq 1 \qquad \forall m \in \mathcal{B}$$

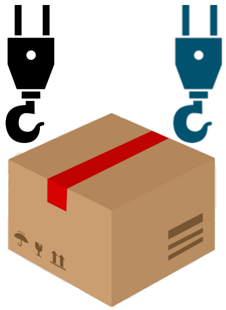
Stacking Constraints

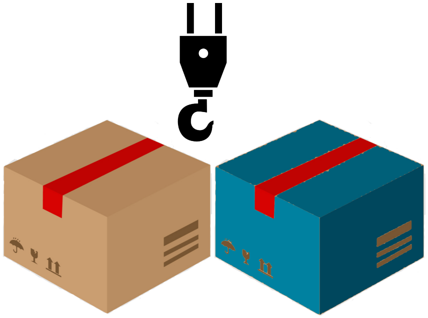


$$x_{pm} \leq \sum_{q \in \mathcal{P}_i} x_{qn} \qquad \forall p \in \mathcal{P}_i : p \neq q, m \in \mathcal{B}, n \in \mathcal{U}_m$$

$$n_{qp} \geq x_{pm} + x_{qn} - 1 \qquad \forall p, q \in \mathcal{P}_i : p \neq q, m \in \mathcal{B}, n \in \mathcal{U}_m$$

Cranes' Constraints





$$\sum_{c \in \mathcal{C}} y_{pc} = 1 \qquad \forall p \in \mathcal{P}_i \cup \mathcal{P}_o$$

$$y_{qc} \leq 1 + n_{pq} + n_{qp} - y_{pc} \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q, \forall c \in \mathcal{C}$$

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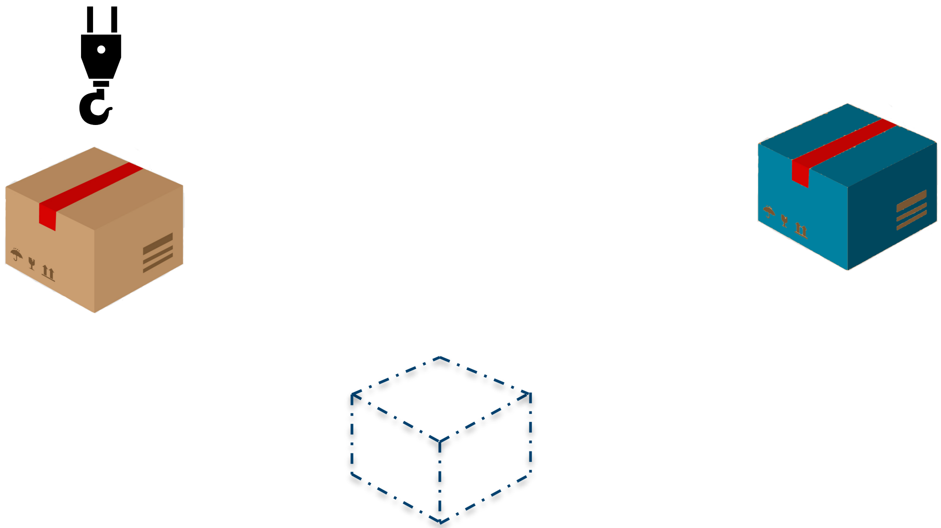
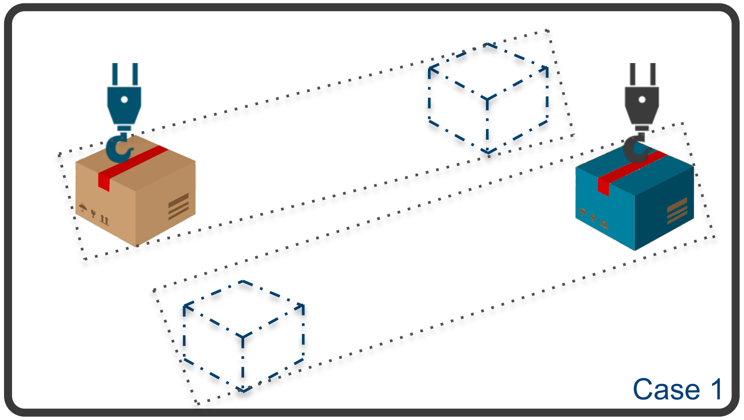
Sequencing movements with the same crane

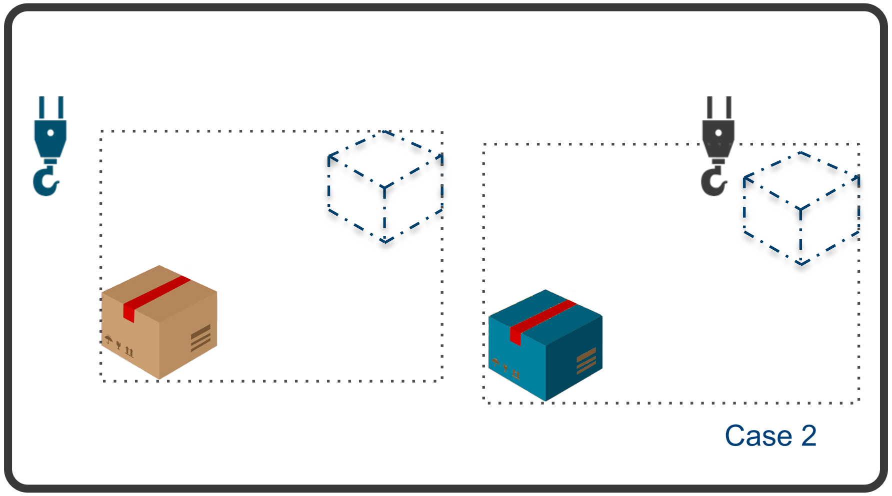


$$s_q \geq s_p + d_p + t_{f_p o_q} - [3 - y_{pc} - y_{qc} - n_{pq}]M$$
$$\forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o, p \neq q, \forall c \in \mathcal{C}$$

Conflicted movements



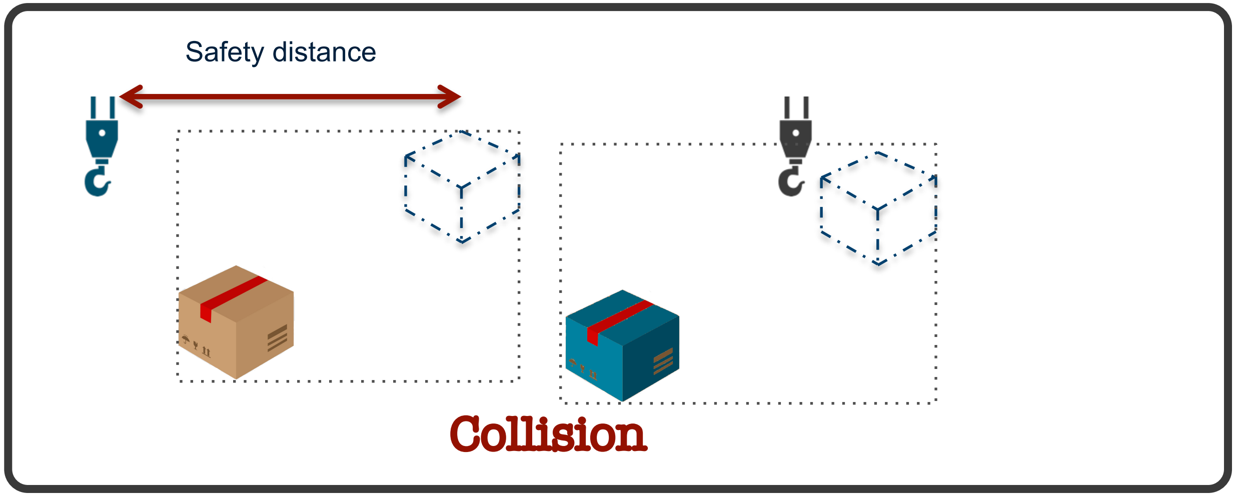
$$l_p - st \geq r_q - o^a_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$
$$l_q - st \geq r_p - o^b_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$



$$r_p \geq l_q - o^c_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$
$$\sum_{c \in \mathcal{C}} c y_{qc} \geq \sum_{c \in \mathcal{C}} c y_{pc} - o^d_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$

$$r_q \geq l_p - o^e_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$
$$\sum_{c \in \mathcal{C}} c y_{pc} \geq \sum_{c \in \mathcal{C}} c y_{qc} - o^f_{pq} M \qquad \forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o : p \neq q$$

Sequencing movements with different crane



$$s_q \geq s_p + g_{pq} + st - [3 - y_{pc} - y_{qc'} - o_{pq} + n_{qp}]M$$
$$\forall p, q \in \mathcal{P}_i \cup \mathcal{P}_o, c, c' \in \mathcal{C}, p \neq q, c \neq c'$$

g_{pq} The waiting time between movement p and q

How to calculate the waiting time?

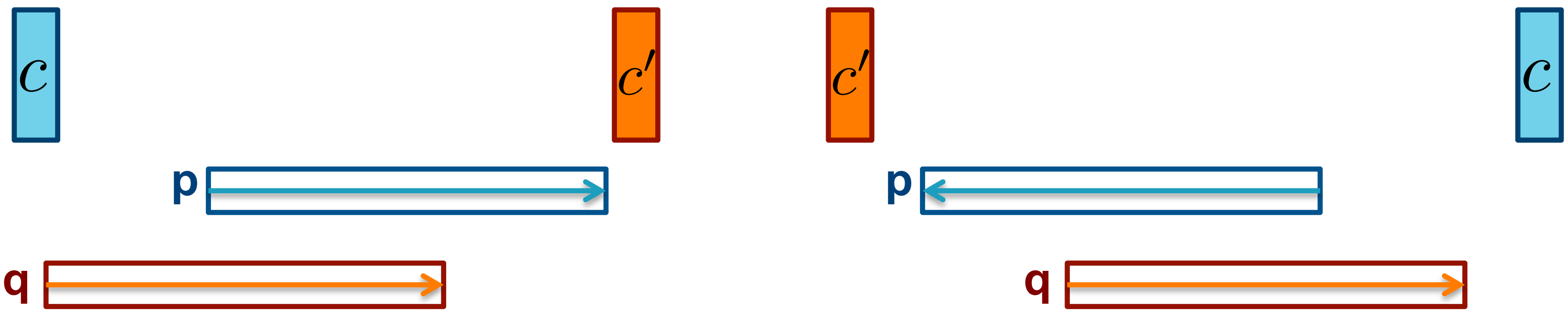
$c' > c$

$xcor_{f_p} > xcor_{o_q}$

or

$c' < c$

$xcor_{f_p} < xcor_{o_q}$



$$g_{pq} = t_{o_p f_p} + t_{f_p o_q} \qquad \forall p \in \mathcal{P}_o$$

$$s_q \geq s_p + g_{pq} + st$$

How to calculate the waiting time?

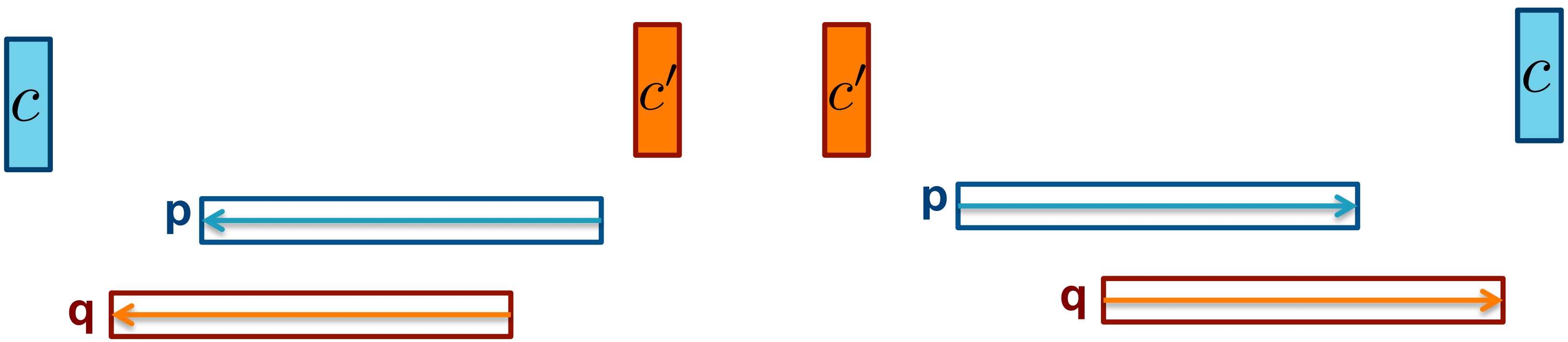
$c' > c$

$xcor_{f_p} < xcor_{o_q}$

or

$c' < c$

$xcor_{f_p} > xcor_{o_q}$

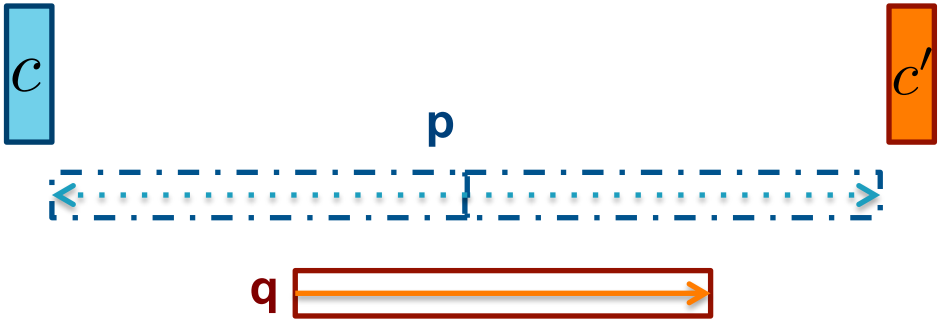


$$g_{pq} = \quad t_{o_p o_q} \qquad \forall p \in \mathcal{P}_o$$

$$s_q \geq s_p + g_{pq} + st$$

How to calculate the waiting time?

For the input products the destination of the movement is not determined



$$g_{pq} = \begin{cases} \displaystyle\sum_{m \in \mathcal{B}: xcor_m > xcor_{o_q}} (t_{o_p m} x_{pm} + t_{m o_q} x_{pm}) + \sum_{m \in \mathcal{B}: xcor_m < xcor_{o_q}} t_{o_p o_q} x_{pm} \\ \forall p \in \mathcal{P}_i, c' > c \\ \displaystyle\sum_{m \in \mathcal{B}: xcor_m < xcor_{o_q}} (t_{o_p m} x_{pm} + t_{m o_q} x_{pm}) + \sum_{m \in \mathcal{B}: xcor_m > xcor_{o_q}} t_{o_p o_q} x_{pm} \\ \forall p \in \mathcal{P}_i, c' < c \end{cases}$$

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Computational Experiments

Instances

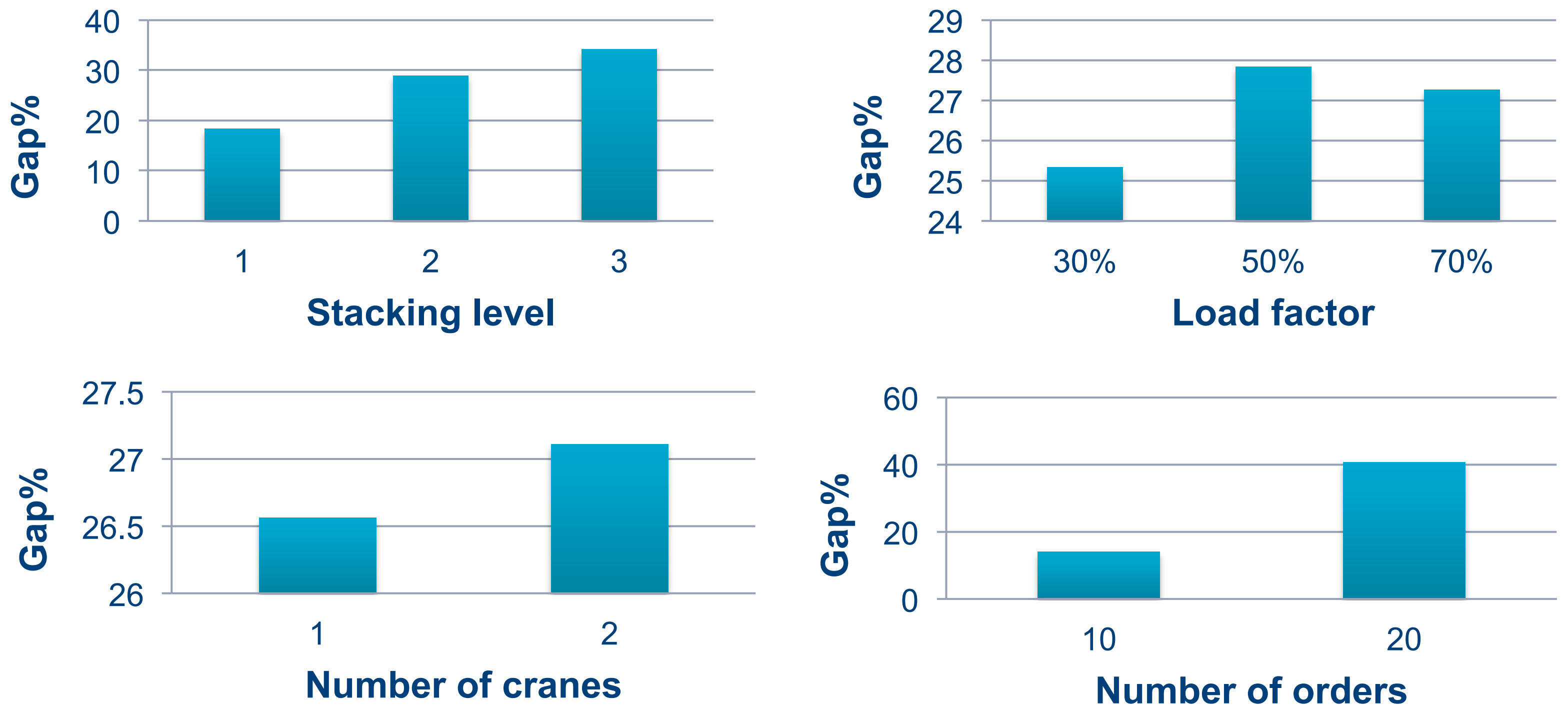
- Yard size = 25 * 10 blocks

Load factor	Number of cranes	Stacking level	Order rate
3	2	3	2

- 108 randomly generated instances
- 200 seconds time limit for model

Some preliminary results

Optimality was achieved for 18 instances



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Conclusion

- Summary
  - A continuous-time model
  - Place assignment + sequencing the orders
  - Various realistic operational constraints:
    - Simultaneous retrieval and storage of products;
    - Dynamic assignment of cranes to operations.
    - Multiple stacking configurations;
    - Multiple input and output points;

- Future Work
  - Develop local search algorithms
  - Matheuristics

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Mathematical formulation for scheduling rail-mounted gantry cranes in warehouses

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